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Third Semester B.E. Degree Examination, December 2011

Advanced Mathematics - I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Express $\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$ in the form $a + ib$. (06 Marks)
- b. Find the modulus and amplitude of $\frac{(3 - \sqrt{2}i)^2}{1 + 2i}$. (07 Marks)
- c. Find the real part of $\frac{1}{1 + \cos\theta + i \sin\theta}$. (07 Marks)
- 2 a. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$. (06 Marks)
- b. If $y = (\sin^{-1} x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$. (07 Marks)
- c. Find the n^{th} derivative of $\frac{x+2}{x+1} + \log\left(\frac{x+2}{x+1}\right)$. (07 Marks)
- 3 a. State and prove Euler's theorem. (06 Marks)
- b. Given $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$, $y = t^2$, find $\frac{du}{dt}$ as a function of t . (07 Marks)
- c. If $x = r \cos\theta$, $y = r \sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$. (07 Marks)
- 4 a. Find the angle of intersection of the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$. (06 Marks)
- b. Find the pedal equation of the curve $\frac{2a}{r} = 1 - \cos\theta$. (07 Marks)
- c. Expand $e^{\sin x}$ by Maclaurin's series upto the term containing x^4 . (07 Marks)
- 5 a. Obtain the reduction formula for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$ where n is a positive integer. (06 Marks)
- b. Evaluate: $\int_1^5 \int_1^{x^2} x(x^2 + y^2) \, dx \, dy$. (07 Marks)
- c. Evaluate: $\int_0^1 \int_0^2 \int_0^2 x^2 yz \, dx \, dy \, dz$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 6 a. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (06 Marks)
- b. Show that $\Gamma(n) = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx$. (07 Marks)
- c. Express $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$ in terms of Gamma function. (07 Marks)
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- 7 a. Solve : $\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$. (06 Marks)
- b. Solve : $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$. (07 Marks)
- c. Solve : $(x^2 - ay)dx = (ax - y^2)dy$. (07 Marks)
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- 8 a. Solve : $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$. (06 Marks)
- b. Solve : $(D - 2)^2 y = 8(e^{2x} + \sin 2x)$. (07 Marks)
- c. Solve : $(D^3 + 4D)y = \sin 2x$. (07 Marks)

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